CHAPTER 5: FILTERS

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3.1 INTRODUCTION

• Is a device that removes or filters unwanted signal.
• Are used to block or pass a specific range of frequencies.

CATEGORIES OF FILTER

PASSIVE FILTER

ACTIVE FILTER

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### 5.1 INTRODUCTION (cont...)

#### DIFFERENCE BETWEEN PASSIVE FILTER & ACTIVE FILTER

<table>
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<tr>
<th><strong>PASSIVE FILTERS</strong></th>
<th><strong>ACTIVE FILTERS</strong></th>
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<tr>
<td>Do not contain an amplifying device.</td>
<td>Contain some type of amplifying device.</td>
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<tr>
<td>Use resistors, capacitors and / or inductors.</td>
<td>Use a transistor and / or operational amplifier in combination with resistors and capacitors to obtain the desired filtering effect.</td>
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<tr>
<td>Always has insertion loss</td>
<td>Have very low insertion loss</td>
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WHAT IS INSERTION LOSS?

• Passive filters never have a power output equal to or greater than the input because the filter always has insertion loss.

• This loss represents the difference between the power received at the load before insertion (installation) of a filter and the power after insertion.

• Is stated as the log of a ratio of power output to power input and is the result of power loss because of resistance in the circuit.
5.2 PASSIVE FILTERS

• Use only passive components such as inductors, capacitors and resistors.
• To minimize distortion in the filter characteristic, it is desirable to use inductors with high quality factors.
• However these are difficult to implement at frequencies below 1 kHz.
• Disadvantages:
  – They are particularly non-ideal (lossy)
  – They are bulky and expensive
5.2 PASSIVE FILTERS (cont…)

**TYPES**

- Low-pass filter
- High-pass filter
- Band-pass filter
- Band-stop filter
5.2 PASSIVE FILTERS (cont...)

Four types of filters - “Ideal”

- **lowpass**
- **highpass**
- **bandpass**
- **bandstop**
5.2 PASSIVE FILTERS (cont...)

Realistic Filters:

- **lowpass**
- **highpass**
- **bandpass**
- **bandstop**
INTERACTIVE LEARNING

Ideal Filters


ideal filter.swf
5.2.1 LOW-PASS FILTER

- Is a circuit that has a constant output voltage up to a cut-off or critical frequency, \( fc \).

- The frequencies above \( fc \) are effectively shorted to ground and are described as being in the attenuation band, or stop-band.

- The frequencies below \( fc \) are said to be in the pass-band.

- Designed using combinations of resistors, inductors and capacitors.
5.2.1 LOW-PASS FILTER (cont...)

**RC LOW PASS FILTER**
- At low frequencies, $X_c$ is greater than $R$, and most of the input signal will appear at the output.
- The cutoff frequency occurs when $X_c = R$

**RL LOW PASS FILTER**
- The inductor is shorted at low frequencies, and most of the output voltage of the circuit dropped across $R$.
- At high frequencies the input acts as an open, and the majority of the input signal is dropped across $L$.
- RL circuit passes low frequencies and blocks high frequencies
5.2.1 LOW-PASS FILTER (cont...)

- Response curve for low-pass filter
- Cutoff frequency

\[ fc = \frac{1}{2\pi RC} \]
\[ fc = \frac{R}{2\pi L} \]

- When output voltage is 0.707 of the input, the power output is 50% of the input power.
- This condition exists at \( fc \) (cutoff frequency)
- If \( P_{out} = \frac{1}{2} P_{in} \), then a 3 dB loss of signal has occurred.

Passes low frequencies
Attenuates high frequencies
5.2.1 LOW-PASS FILTER (cont...)

- Have a wide variety of applications in electronic circuits.
- One common application is in smoothing the output of a pulsating DC signal.
- The action of capacitor in an RC circuit is to resist a change in voltage across the capacitor.
- If the direct current in the circuit has a ripple, the action of the RC circuit is to suppress or absorb the variations and to delivered a filtered DC current and voltage to the load.

A block diagram of a low-pass filter and rectifier
Example 1
Determine the cutoff frequency of the circuit.
Answer: 7.23 kHz

Example 2
For the circuit, calculate the half-power frequency.
Answer: 52.52 kHz
5.2.2 HIGH-PASS FILTER

- Allows frequencies above the cutoff frequency to appear at the output, but blocks or shorts to ground all those frequencies below the -3dB point on the curve.
5.2.2 HIGH-PASS FILTER (cont...)

**RC HIGH-PASS FILTER**
- Capacitive reactance is large at low frequencies and small at high frequencies.
- The capacitor acts as a coupling capacitor.

**RL HIGH-PASS FILTER**
- There is a high value of inductive reactance at high frequencies. The high frequencies produce a large output voltage across the inductor.
- At low frequencies, $X_L$ will be small and the low frequencies will be effectively shorted to ground.
- It should be noted that RL filters are not often used in active circuits because of their size, weight, and losses.

![RC High-Pass Filter Circuit](image1)

![RL High-Pass Filter Circuit](image2)
5.2.2 HIGH-PASS FILTER (cont...)

The cutoff frequency on the high-pass response curve is called the low cutoff frequency.

Response curve for high-pass filter

Cutoff frequency

\[ f_c = \frac{1}{2\pi RC} \]
\[ f_c = \frac{R}{2\pi L} \]

Passes high frequencies
Attenuates low frequencies
Example 3
Design an RL high-pass filter that will have a cutoff frequency of 1500 Hz.
Assume $R = 1 \, k\Omega$

Answer : $106.1 \, mH$
5.2.3 BAND-PASS FILTER

- Allows a certain range or band of frequencies to pass through it relatively unattenuated.
- Designed to have a very sharp, defined frequency response.
- Equivalent to combining a low-pass filter and a high-pass filter.
- The impedance of the circuit increases as the frequencies increase and decrease from the resonant frequency.
- Only the frequencies close to resonance are passed.
- All others are blocked because of high circuit impedance.
5.2.3 BAND-PASS FILTER (cont...)

Series resonant band-pass filter

Parallel resonant band-pass filter

Resonant frequency

\[ fr = \frac{1}{2\pi\sqrt{LC}} \]
The cutoff frequency of the high pass section becomes the lower frequency limit in the passband $f_1$.

The upper frequency in the passband $f_2$, is the result of the cutoff frequency in the low pass section.

The passband, or bandwidth, is the difference between $f_2$ and $f_1$.

At resonant frequency $f_r$, $X_L = X_C$, and the output voltage is equal to the input signal.
• Depending on the design and purpose of the band-pass filter, the bandwidth may be very wide or very narrow.

• Some of common uses of band-pass filters are:
  i. Audio – to equalize sound levels
  ii. Communications – to select a narrow band of radio frequencies.
  iii. Audio – to produce speaker crossover networks.
Example 4
For the circuit, calculate the resonant frequency and bandwidth.

Solution

\[ fr = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{(100\,\text{mH})(0.1\,\mu\text{F})}} = 1591.55\,\text{Hz} \]

For series resonant frequencies

\[ Q = \left( \frac{1}{R} \right) \sqrt{\frac{L}{C}} = \left( \frac{1}{100\,\Omega} \right) \sqrt{\frac{100\,\text{mH}}{0.1\,\mu\text{F}}} = 10 \]

\[ BW = \frac{fr}{Q} = \frac{1591.55\,\text{Hz}}{10} = 159.16\,\text{Hz} \]
5.2.3 BAND-PASS FILTER (cont...) 

**Example 5**
A parallel resonant band-pass filter has a lower cutoff frequency of 6.3 kHz and an upper cutoff frequency of 8 kHz. Determine the bandwidth and resonant frequency

**Solution**

\[ BW = f_{c1} - f_{c2} = 8\text{kHz} - 6.3\text{kHz} = 1.7\text{kHz} \]

\[ fr = \sqrt{f_{c1} f_{c2}} = \sqrt{(8\text{kHz})(6.3\text{kHz})} = 7.1\text{kHz} \]
5.2.4 BAND-STOP FILTER

• Rejects signals at frequencies within a special band and passes signals at all other frequencies.
• Also referred to as a band-reject, notch, wave-trap, band-elimination, or band-suppression filter.
• Like the band-pass filter, the band-stop filter may also be formed by combining a low-pass and high-pass filter.

The band-stop filter is designed so that the cutoff frequency of the low-pass filter is below that of the high-pass filter.
5.2.4 BAND-STOP FILTER (cont…)

**SERIES RESONANT BAND-STOP FILTER**

- When a series resonant LC circuit is connected in parallel with a load resistor, the LC network provides a low impedance shunt path.
- At resonance, the LC circuits acts as a short circuit and the signal does not pass through R.
- At frequencies above or below resonance, the LC circuit offer high resistance and current flow through the load.

**PARALLEL RESONANT BAND-STOP FILTER**

- When the parallel LC network is at resonance, the impedance is very high and virtually no current flows to the load.
- The impedance decrease above and below resonance, so all other signals are passed through the filter.
5.2.4 BAND-STOP FILTER (cont...)

- Popular in variety of applications, particularly audio electronics.
- These filters can be used in groups to provide equalization of the entire audio spectrum.
- For example, a typical one-third octave equalizer will contain 28 band-rejection filters.
- Each filter section is designed to remove a specific band of frequencies and provides up to 15 dB attenuation at its resonant frequency.
**Example 6**
For the circuit, calculate the resonant frequency. Given $L = 200 \, \text{mH}$ and $C = 0.01 \, \mu\text{F}$.

**Solution**

$$fr = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(200\,\text{mH})(0.01\,\mu\text{F})}} = 3.56\,\text{kHz}$$
5.3 ACTIVE FILTERS

• Use active components or devices such as transistors and op-amps, that can amplify.
• First order filter – use one resistor and one reactive component, which is either an inductor or capacitor.
• Second order filter – use two resistors and two reactive components.
5.3 ACTIVE FILTERS (cont...)

- Low-pass filter
- High-pass filter
- Band-pass filter
- Band-stop filter
5.3 ACTIVE FILTERS (cont…)

- Advantages of active filters include:
  - reduced size and weight, and therefore parasitic
  - increased reliability and improved performance
  - simpler design than for passive filters and can realize a wider range of functions as well as providing voltage gain
  - in large quantities, the cost of an IC is less than its passive counterpart
Active filters also have some disadvantages:

- limited bandwidth of active devices limits the highest attainable pole frequency and therefore applications above 100 kHz (passive RLC filters can be used up to 500 MHz)
- the achievable quality factor is also limited
- require power supplies (unlike passive filters)
- increased sensitivity to variations in circuit parameters caused by environmental changes compared to passive filters

For many applications, particularly in voice and data communications, the economic and performance advantages of active filters far outweigh their disadvantages.
5.3.1 ACTIVE LOW-PASS FILTER

- At low frequencies, the capacitor has an extremely high capacitive reactance.
- Therefore, the circuit acts as an inverting amplifier with a voltage gain of $-\frac{R_f}{R_1}$.
- At higher frequencies, the capacitive reactance of $C_f$ decreases.
- This causes the voltage gain to decrease due to the increased amount of negative feedback.
- When $X_{c_f} = R_f$, the voltage gain equals 70.7% of its midband value.
- This is the cutoff frequency $f_c$:

$$f_c = \frac{1}{2\pi R_f C_f}$$
5.3.1 ACTIVE LOW-PASS FILTER
(cont...)

- The voltage gain for any frequency is calculated as

\[ Av = - \frac{Z_f}{R_1} \]

where

\[ Z_f = \frac{Xc_f R_f}{\sqrt{R_f^2 + Xc_f^2}} \]

- The voltage gain in decibels

\[ Av_{(dB)} = 20 \log \frac{Z_f}{R_1} \]
**5.3.1 ACTIVE LOW-PASS FILTER**

*(cont...)*

**Example 7**

Refer to figure:

i) Calculate the cutoff frequency, $f_c$

ii) Calculate the voltage gain at (a) 0 Hz and (b) 1 MHz

iii) Calculate the dB voltage gain at (a) 0 Hz and (b) 1.591 kHz.
5.3.1 ACTIVE LOW-PASS FILTER

(cont...)

Solution

i) \( f_c = \frac{1}{2\pi R_f C_f} = \frac{1}{2\pi (10k\Omega)(0.01\mu F)} = 1.591kHz \)

ii) (a) At 0 Hz, \( X_c f \approx \infty \Omega \). Therefore, the voltage gain is

\[
Av = -\frac{R_f}{R_1} = -\frac{10k\Omega}{1k\Omega} = -10
\]

(b) At 1 MHz,

\[
X_c f = \frac{1}{2\pi f_c C_f} = \frac{1}{2\pi (1MHz)(0.01\mu F)} = 15.9\Omega
\]

\[
Z_f = \frac{X_c f R_f}{\sqrt{R_f^2 + X_c f^2}} = \frac{(15.9\Omega)(10k\Omega)}{\sqrt{10k\Omega^2 + 15.9\Omega^2}} = \frac{159k\Omega}{10k\Omega} = 15.9\Omega
\]

\[
Av = -\frac{R_f}{R_1} = -\frac{15.9\Omega}{1k\Omega} = -0.0159
\]
5.3.1 ACTIVE LOW-PASS FILTER
(cont...)

Solution

iii) (a) At 0 Hz, $X_c \approx \infty \Omega$. Therefore, $Z_f = R_f$

$$Av_{(dB)} = 20 \log \frac{R_f}{R_1} = 20 \log \frac{10k\Omega}{1k\Omega} = 20dB$$

(b) The frequency 1.591 kHz is the cutoff frequency, $f_c$. At this frequency,

$$Z_f = 0.707 \cdot R_f = 0.707 \cdot (10 \ k\Omega) = 7.07 \ k\Omega.$$  

$$Av_{(dB)} = 20 \log \frac{Z_f}{R_1} = 20 \log \frac{7.07k\Omega}{1k\Omega} = 17dB$$

Notice that $Av$ is down 3 dB from its pass-band value of 20 dB
At high frequencies, $Xc_1 \approx 0 \, \Omega$ and the voltage gain $Av$, equals $-\frac{R_f}{R_1}$.

At very low frequencies, $Xc_1 \approx \infty \, \Omega$ and the voltage gain $Av$ approaches zero.

The cutoff frequency $fc$, is given by:

$$fc = \frac{1}{2\pi R_1 C_1}$$

At $fc$, the voltage gain $Av$, is at 70.7% of its midband value.

The voltage gain for any frequency is calculated as

$$Av = -\frac{R_f}{Z_1} \quad \text{where} \quad Z_1 = \sqrt{R_1^2 + Xc_1^2}$$

Voltage gain in decibels

$$Av_{(dB)} = 20 \log \frac{R_f}{Z_1}$$
5.3.2 ACTIVE HIGH-PASS FILTER
(cont...)

Example 8
Refer to figure, calculate the cutoff frequency, \( f_c \).
Given \( C_1 = 0.1 \, \mu F \) and \( R_1 = 1 \, k\Omega \)

Solution

\[
fc = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi (1k\Omega)(0.1\mu F)} = 1.591 kHz
\]
5.3.3 ACTIVE BAND-PASS FILTER

• Besides low-pass and high-pass filters, other common types are band-pass filter (rejects high and low frequencies, passing only signal around some intermediate frequency).

• The simplest band-pass filter can be made by combining the first order low pass and high pass filters that we just looked at.
5.3.3 ACTIVE BAND-PASS FILTER (cont…)

- This circuit will attenuate low frequencies ($\omega << 1/R_2 C_2$) and high frequencies ($\omega >> 1/R_1 C_1$), but will pass intermediate frequencies with a gain of $-R_1/R_2$.

- However, this circuit cannot be used to make a filter with a very narrow band. To do that requires a more complex filter.
5.3.3 ACTIVE BAND-PASS FILTER (cont...)

- Figure below shows a band-pass filter using two stages, the first a high-pass filter and the second a low-pass filter, the combined operation being the desired band-pass response.
Example 9
Calculate the cutoff frequencies of the band-pass filter circuit of figure in the slide before with $R_1 = R_2 = 10 \, k\Omega$, $C_1 = 0.1 \, \mu F$, and $C_2 = 0.002 \, \mu F$.

Solution

\[
\begin{align*}
  f_{LP} &= \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi (10k\Omega)(0.1\mu F)} = 159.15 \, Hz \\
  f_{HP} &= \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi (10k\Omega)(0.002\mu F)} = 7.96 \, kHz
\end{align*}
\]
In analyzing filters, the decibel (dB) unit is often used to describe the amount of attenuation offered by the filter.

In basic terms, the decibel is a logarithmic expression that compares two power levels.

The power gain or loss in decibels can also be computed from a voltage ratio if the measurement are made across equal resistance.

\[ N_{dB} = 20 \log \frac{V_{out}}{V_{in}} \]

Where
- \( N_{dB} \) = gain or loss in decibels
- \( V_{in} \) = input voltage
- \( V_{out} \) = output voltage
Example 9
In figure below, calculate the attenuation, in decibels, at the following frequencies: (a) 0 Hz, (b) 1.592 kHz, (c) 15.92 kHz.
Assume that \( V_{in} = 10 \, V_{pp} \) at all frequencies.

Solution:

a) At 0 Hz, \( V_{out} = V_{in} = 10 \, V_{pp} \),
   since the capacitor \( C \) appears as an open.
   Therefore,

   \[
   N_{dB} = 20 \log \frac{P_{out}}{P_{in}} = 20 \log \frac{10V_{pp}}{10V_{pp}} = 20 \log 1 = 0\, dB
   \]
**5.4 DECIBELS & FREQUENCY RESPONSE CURVE** (cont...)

**Solution:**

b) Since 1.592 kHz is the cutoff frequency \( f_c \), \( V_{out} \) will be 0.707 \( (V_{in}) \) or 0.707 \( V_{pp} \). Therefore

\[
N_{dB} = 20 \log \left( \frac{P_{out}}{P_{in}} \right) = 20 \log \left( \frac{7.07V_{pp}}{10V_{pp}} \right) = 20 \log 0.707 = -3dB
\]

c) To calculate \( N_{dB} \) at 15.92 kHz, \( X_c \) and \( Z_T \) must first determined.

\[
X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi (15.92kHz)(0.01\mu F)} = 1k\Omega
\]

\[
Z_T = \sqrt{R^2 + X_c^2} = \sqrt{10^2k\Omega + 1^2 k\Omega} = 10.05k\Omega
\]

\[
N_{dB} = 20 \log \left( \frac{X_c}{Z_T} \right) = 20 \log \left( \frac{1k\Omega}{10.05k\Omega} \right) = 20 \log 0.0995 = -20dB
\]
In Example 9, notice that $N_{dB}$ is 0 dB at a frequency of 0 Hz, which is in the filter’s pass-band.

This may seem unusual, but the 0 dB value simply indicates that there is no attenuation at this frequency.

For an ideal passive filter, $N_{dB} = 0$ dB in the pass-band.

As another point of interest, $N_{dB}$ is -3 dB at the cutoff frequency of 1.592 kHz.

Since $V_{out} = 0.707 V_{in}$ at $fc$ for any passive filter, $N_{dB}$ is always -3 dB at the cutoff frequency of a passive filter.

The $N_{dB}$ value of loss can be determined for any filter if the values of $V_{in}$ and $V_{out}$ are known.
"Dunia tercipta bila kita berusaha"